Implications of primordial black holes on the first stars and the origin of the super–massive black holes

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Abstract

If the cosmological dark matter has a component made of small primordial black holes, they may have a significant impact on the physics of the first stars and on the subsequent formation of massive black holes. Primordial black holes would be adiabatically contracted into these stars and then would sink to the stellar center by dynamical friction, creating a larger black hole which may quickly swallow the whole star. If these primordial black holes are heavier than $\sim 10^{22}$ g, the first stars would likely live only for a very short time and would not contribute much to the reionization of the universe. They would instead become $10 - 10^3\ M_\odot$ black holes which (depending on subsequent accretion) could serve as seeds for the super–massive black holes seen at high redshifts as well as those inside galaxies today.

1 Introduction

The first stars in the Universe mark the end of the cosmic dark ages, reionize the Universe, and provide the enriched gas required for later stellar generations. They may also be important as precursors to black holes (BHs) that coalesce and power bright early quasars. The first stars are thought to form inside Dark Matter (DM) halos of mass $10^5\,M_\odot$–$10^6\,M_\odot$ at redshifts $z = 10 - 50$ (Abel et al. (2002); Bromm et al. (2002); Yoshida et al. (2003)). These halos consist of 85% DM and 15% baryons in the form of metal-free gas made of H and He. Theoretical calculations indicate that the baryonic matter cools and collapses via H_2 cooling (Peebles & Dicke (1968); Matsuda et al. (1971); Hollenbach & McKee (1979); Tegmark et al. (1997)) into a single small protostar (Omukai & Nishi (1998)) at the center of the halo (for reviews see Ripamonti & Abel (2005), Barkana & Loeb (2001) and Bromm & Larson (2004)).

It is interesting to study the effects on the evolution of the first stars due to the large reservoir of DM within which these stars form. The first protostars and stars are particularly good sites for this investigation because they form inside the highest density environment (compared to today's stars): they form at high redshifts (density scales as $(1+z)^3$) and in the high density centers of DM haloes. Previously, two of us (Spolyar et al.(2008)) studied the effects of Weakly Interacting Massive Particles on the first stars: we found that the annihilation of these particles could provide a heat source for the star that stops the collapse of the protostar as well as potentially dominates over any fusion luminosity for a long time.

In this paper, we consider instead the effects on these first stars of a different candidate for the DM: Primordial Black Holes (PBHs). These are small black holes that may be formed in the very early universe (see the next section for more detail) and may exist in sufficient abundance to provide the DM seen in the Universe today. The masses of PBHs that explain the entirety of the DM range from $(10^{17} - 10^{26})$ g; heavier PBHs up to $1\,M_\odot$ may provide still an interesting fraction of the DM.

We discuss the implications that PBH DM would have on the physics of the first stars, the so called Population III stars. These stars could range from $\sim$ (1 - few 100) $M_\odot$. First, we compare various possible heat sources due to PBHs with the ordinary heat from stellar fusion of the stars. For the properties of the Pop III stars, we use results computed by Heger & Woosley. Specifically, for a 100 $M_\odot$ (10 $M_\odot$) Pop. III star, we take the central temperature to be 1.2×10^8 K (9.6×10^7 K), the central density 31 g/cm^3 (226 g/cm^3), the radius 7 $R_\odot$ (1.2 $R_\odot$), and the stellar fusion luminosity to be

$$L_* = 6.5 \times 10^{39}\,\mathrm{erg/s} \quad (100\,M_\odot)\,, \tag{1}$$
$$L_* = 4.2 \times 10^{37}\,\mathrm{erg/s} \quad (10\,M_\odot)\,. \tag{2}$$

We find that the ordinary stellar fusion luminosity dominates over the heat sources due to PBHs, which include accretion onto the BHs, Hawking radiation, and the Schwinger mechanism.

Instead, we find the interesting result that PBHs inside the first stars may sink to the center and form a single BH, which may accrete very rapidly and swallow the whole star. The phenomenon is relevant for PBHs heavier than about 10^{22} g, because the corresponding timescale for dynamical friction turns out to be shorter than the typical stellar lifetime, while it is less interesting or completely negligible for lighter BHs. So, for $M_{PBH} \gtrsim 10^{22}$ g, the lifetimes of Pop. III stars may be shortened, with implications for reionization of the Universe as well as for the first supernovae. In addition, since the stars are inside much larger haloes, they can in principle accrete even more matter (depending on the accretion mechanism). Thus, the end–products of the scenario are BHs of masses $10 - 10^5\,M_\odot$. These may be the seeds which produced the super–massive BHs seen at high redshifts; the Intermediate Mass Black Holes; as well as the black holes at the center of every normal galaxy today and whose origin is as yet uncertain. Possible mechanisms of production of superheavy BHs are reviewed in Dokuchaev et al. (2007). In addition, although the PBH swallowing the star shortens the star's lifetime and its contribution to reionization, the newly formed hole can become a new, alternative source of ionizing photons.

The rest of the paper is organized as follows. In Section II, we briefly review the physics of PBHs, that is, how they can be formed in the early Universe and what current constraints on their cosmological abundance are. In Section III, we discuss the behavior of individual PBHs: how many

of them are expected inside a single star (via adiabatic contraction), what is the luminosity due to accretion onto the PBHs, and what is the timescale for their size to double. We also investigate alternative mechanisms for generating luminosity by these small PBHs. Then in Section IV and V we turn to the most important part of the paper. We study the dynamical friction that pulls all the BHs into a single larger BH at the center of the star, and then watch this single large BH accrete the entire star surrounding it on a fairly rapid timescale. We conclude with a discussion in Section VI. Throughout the paper, we use units with $c = 1$.

2 Physics of Primordial Black Holes

2.1 Production mechanisms

PBHs may be formed in the early universe by many processes (Zeldovich & Novikov (1966), Hawking (1971), Carr & Hawking (1974), Crawford & Schramm (1982), Hawking (1989), Polnarev & Zembowicz (1991), Dolgov & Silk (1993), Jedamzik (1997), Rubin et al. (2000), Dolgov et al. (2008)). For a general review, see e.g. Carr (2003). The earliest mechanism for BH production can be fluctuations in the space-time metric at the Planck epoch. Large number of PBHs can also be produced by nonlinear density fluctuations due to inhomogeneous baryogenesis at small scales (Dolgov & Silk (1993), Dolgov et al. (2008)). If within some region of space density fluctuations are large, so that the gravitational force overcomes the pressure, we can expect the whole region to collapse and form a BH. In the early Universe, generically, BHs of the horizon size are formed, although it is also possible to form much smaller BHs (Polnarev & Zembowicz (1991), Hawking (1989)). BHs can also be produced in first and second order phase transitions in the early Universe (Crawford & Schramm (1982), Jedamzik (1997)). Gravitational collapse of cosmic string loops (Polnarev & Zembowicz (1991), Hawking (1989)) and closed domain walls (Rubin et al. (2000)) can also yield BHs. The masses of PBHs formed in the above mentioned processes range roughly from M_{Pl} (BHs formed at the Planck epoch) to $M_\odot$ (BHs formed at the QCD phase transition).

The basic picture is that energy perturbations of order one stopped expanding and recollapsed as soon as they crossed into the horizon (Zeldovich & Novikov (1966), Hawking (1971), Carr & Hawking (1974)). The maximal mass of PBHs is set by the total mass within the cosmological horizon, i.e. $M_{hor} = M_{pl}^3/\Lambda^2$ at any given energy scale Λ at which the BH forms. This is also the expected mass scale of a BH in most early Universe scenarios for the production of PBHs (it can be at most a factor of 10^{-4} smaller (Hawke & Stewart (2002))). Thus

$$M_{PBH} \approx \frac{t_f}{G_N} \approx 5 \cdot 10^{26}\, g_*^{-1/2} \left(\frac{1\ \mathrm{TeV}}{T_f}\right)^2\ \mathrm{g}\,, \tag{3}$$

where we assumed a radiation dominated Universe, with g_* the effective number of relativistic degrees of freedom and T_f the temperature of the Universe at time t_f.

2.2 Observational constraints

PBHs in the mass range $M_{PBH} \sim 10^{17} - 10^{26}$ g can be good DM candidates. A number of constraints restrict the mass to this range. PBHs with an initial mass smaller than about $5 \cdot 10^{14}$ g are expected to be already evaporated due to Hawking radiation; moreover their presence in the early Universe can be constrained by observations for $M_{BH} \gtrsim 10^9$ g (lifetime $\tau \gtrsim 1$ s) (Novikov et al. (1979)). For $M_{PBH} \sim 10^{15}$ g, there are strong bounds as well, at the level of $\Omega_{PBH} \lesssim 10^{-8}$, from the observed intensity of the diffuse gamma ray background (Page & Hawking (1976)), so they may be at most a tiny fraction of the non–relativistic matter in the Universe. For larger masses, constraints can be deduced from microlensing techniques (Alcock et al. (2000); Tisserand et al. (2007)) and dynamical arguments (Carr & Sakellariadou (1999)), which exclude the possibility that the whole cosmological DM is made of BHs heavier than 10^{26} g, even if they still may be an important component. For example, the PBH to DM mass ratio in the Galactic Halo would be smaller than 0.04 for PBHs in the mass range $10^{30} - 10^{32}$ g and than 0.1 for the mass range $10^{27} - 10^{33}$ g (Tisserand et al. (2007)).

On the other hand, for the mass range $10^{17} - 10^{26}$ g, there are currently no clear observational methods of detection. For $M_{PBH} \sim 10^{17} - 10^{20}$ g, the presence of PBHs can be inferred from the femptolensing of gamma ray burts (Gould (1992), Nemiroff & Gould (1995), Marani et al. (1999)), but the constraint is weak, roughly $\Omega_{PBH} \lesssim 0.2$; in addition it holds only for uniformly distributed DM and is not easy to extend to the more realistic case of clumped DM. The same mass range might be covered by future gravitational wave space antennas, from the gravitational interaction of PBHs with test masses of the laser interferometer (Seto & Cooray (2004)), but the expected detection rate for LISA is too low and only a further generation of space detectors might put non–trivial constraints. According to recent work Abramowicz et al. (2008), the PBH mass range $10^{15} - 10^{26}$ g remains unexplored and thus allowed. However, further constraints raise the lower bound to roughly $10^{16} - 10^{17}$ g (Bambi et al. (2008a)).

We present results for PBHs with mass $M_{BH} = 10^{24}$ g but show the scaling for other PBH masses in the $10^{17} - 10^{26}$ range. Our results are qualitatively the same for PBHs of any mass in the allowed range. For heavier PBHs up to e.g. $1\,M_\odot$, the results will be somewhat different and discussed in the discussion section.

3 Primordial Black Holes inside the Star

In this section we study the behavior of the PBHs inside the star. We estimate the total mass in these objects, as well as the luminosity and timescale for accretion onto individual PBHs.

3.1 Total Mass in PBHs inside the star

The first stars form at the centers of $10^6 M_\odot$ DM haloes. As a starting point we assume an initial Navarro–Frenk–White profile (Navarro et al. (1997)) for both DM (85% of the mass) and baryons (15% of the mass). As the gas collapses to form a star, it gravitationally pulls the DM (in this case PBHs) with it. We use adiabatic contraction (Sellwood & McGaugh (2005)) to find the resultant dark matter profile inside the star (Spolyar et al. (2008))

$$\rho_{DM} \approx 5\,(n_b \text{ cm}^3)^{0.8} \text{ GeV/cm}^3 , \tag{4}$$

which is independent of the nature of DM[1]. Here, n_b is the mean baryon density inside the star. It should be noted that adiabatic contraction is not a relaxation process. Instead as the baryons collapse to form a star, they gravitationally pull the DM will them, so that the DM density inside the star increases. Hence, the DM evolves on the timescale of the baryons. Ideally, instead of using the adiabatic approximation, it would be desirable to run an N-body simulation. At present this is technically not possible. Regardless, adiabatic contraction should give a reasonable approximation and is widely used formalism[2]. In addition, the formal requirements to apply the adiabatic approximation hold during most of the evolution of the baryons. For a mean baryon number density $n_b \approx 10^{24}$ cm^{-3}, the DM to baryon matter mass ratio of a typical Pop. III.1 star is at the level of 10^{-4}. The number of PBHs inside the star is roughly

$$N_{BH} \sim 10^7 \left(\frac{10^{24}\, g}{M_{BH}}\right) \left(\frac{M_*}{100\, M_\odot}\right), \tag{5}$$

where M_* is the mass of the Pop. III star. More precisely (modeling the star as an $n = 3$ polytrope), we find for a 100 $M_\odot$ (10 $M_\odot$) star that the total mass in PBHs is

$$M_{tot,PBH} \quad = \quad 6.3 \times 10^{30} \text{ g} \quad (100\, M_\odot) , \tag{6}$$

[1] This is the result of a calculation for DM density in the first stars that we performed with WIMP dark matter in mind, but exactly the same result holds for any type of DM including BHs which are orders of magnitude larger.

[2] Our original work on adiabatic contraction in the first stars was performed using a very simple assumption of circular orbits only. However, in follow-up work, two of us participated in a paper (Freese et al. (2009)) in which we performed an exact calculation including radial orbits. The results changed by less than a factor of two, so that we feel comfortable using Eq.(4). In that same paper we also considered a core alternative to an NFW profile as our starting point for the adiabatic contraction and, again, obtained essentially the same result. Our results for DM densities in the first stars appear to be quite robust.

$$M_{tot,PBH} = 4.1 \times 10^{29}\ \mathrm{g}\ \ (10\,M_\odot)\,. \tag{7}$$

3.2 Accretion onto the PBHs from stellar material

In this paper we study the effects of PBHs on the stars on the main sequence, once they have fusion proceeding in their cores. The PBH effects are much more important during this stage than during the protostellar collapse phase. Since they are surrounded by a high density stellar environment, they accrete and emit radiation. As a maximum possible value, the accretion luminosity for a single PBH cannot exceed the Eddington limit

$$L_E = \frac{4\pi G_N M_{BH} m_p}{\sigma_{Th}} = 6.5 \cdot 10^{28} \left(\frac{M_{BH}}{10^{24}\ \mathrm{g}}\right)\ \mathrm{erg/s}\,, \tag{8}$$

where σ_{Th} is the Thomson cross section. L_E is the luminosity at which the outwards radiation pressure compensates the gravitational attraction and stops the accretion process. Clearly, the Eddington luminosity is proportionate to mass. In this case, the mass has been conservatively set to the mass of the BH (M_{BH}). In fact, the mass should include the optically thick gas surrounding the BH. Under this restriction, the maximum stellar luminosity from PBHs inside one star is realized when the accretion luminosity of every BH is at the Eddington limit, i.e.

$$L_{E,tot} = N_{BH} L_E \sim 10^{36} \left(\frac{M_*}{100\ M_\odot}\right)\ \mathrm{erg/s}. \tag{9}$$

Since $L_{E,tot} \propto M_{PBH,tot}$, the upper bound on the power emitted by PBHs is independent of the BH mass. This accretion powered luminosity is at least a few orders of magnitude smaller than the expected stellar luminosity for Pop. III stars, $4 \cdot 10^{37}$ erg/s ($6 \cdot 10^{39}$ erg/s) for 10 and 100 $M_\odot$ (Freese et al. (2008)) stars respectively. The extra heat produced by accretion onto the PBHs inside the star has thus a negligible impact on the physics of the star.

As the PBHs accrete more matter and become more massive, the Eddington limit increases and the BH accretion luminosity becomes more and more important in the energy balance of the star. The Bondi accretion rate is (Bondi (1952))

$$\dot{M}_B = 4\pi R_B^2\, \rho_b\, v = 1.4 \cdot 10^{12} \left(\frac{M_{BH}}{10^{24}\ \mathrm{g}}\right)^2 \left(\frac{1\ \mathrm{keV}}{T}\right)^{3/2} \left(\frac{\rho_b}{1\ \mathrm{g/cm^3}}\right)\ \mathrm{g/s}\,. \tag{10}$$

Here $R_B = 2G_N M_{BH}/v^2$ is the Bondi radius. The quantity v is the typical velocity of the particles of the accreting gas with respect to the BH, and should account for both the thermal velocity of the particles $v_p = \sqrt{3T/m_p}$, where T is the local temperature of the star, as well as the BH orbital velocity $v_{BH} = \sqrt{G_N M_*(r)/r}$, where $M_*(r)$ is the stellar mass within a distance r from the center. Close to the center $v \approx v_p$, but for large r this relation is no longer true; instead, the BH orbital velocity may reduce the accretion rate, even by an order of magnitude. We here take $v = v_p$ and use the Bondi formula to find an upper limit on the accretion rate, recognizing that this value may well overestimate the true accretion rate [3]. The differential equation $\dot{M}_{BH} = \alpha M_{BH}^2$ has solution

$$M_{BH}(t) = \frac{M_0}{1 - \alpha M_0 t}\,, \tag{11}$$

where M_0 is the BH mass at $t = 0$ and

$$\alpha M_0 = 1.6 \cdot 10^{-13} \left(\frac{M_{BH}}{10^{24}\ \mathrm{g}}\right) \left(\frac{1\ \mathrm{keV}}{T}\right)^{3/2} \left(\frac{\rho_b}{1\ \mathrm{g/cm^3}}\right)\ \mathrm{s}^{-1} \tag{12}$$

[3] Moreover, the Bondi formula holds in the ideal case of perfect spherical symmetry. In realistic situations, the non–zero angular momentum of the accreting gas and the presence of other effects (magnetic fields, turbulences, etc.) may diminish the accretion rate, since L_a must be smaller than L_E. The case of accretion onto BHs is however a complex phenomenon, because BHs have an event horizon and in principle may be capable of swallowing an arbitrary amount of matter without exceeding the Eddington luminosity (Begelman (1978), Begelman et al. (2008)). We will take the Bondi accretion as an upper limit (Begelman (1978)).

is the inverse of the characteristic accretion time of the BH. The accretion time scale is thus not shorter than

$$\tau_a \sim 10^5 \left(\frac{10^{24}\ \mathrm{g}}{M_{BH}}\right)\left(\frac{T}{1\ \mathrm{keV}}\right)^{3/2}\left(\frac{1\ \mathrm{g/cm}^3}{\rho_b}\right)\ \mathrm{yr}\,. \tag{13}$$

It is possible for even a single PBH with $M_{BH} > 10^{24}$ g inside the star to eat the entire star. Such a case was discussed in Begelman (1978) in the context of a super–massive star capturing a BH in a bound orbit. The current scenario differs due to the fact that we are interested in the role of PBHs on Pop. III stars and their effects on cosmology (e.g. reionization); here the PBHs are thought to comprise at least some measurable fraction of the DM in the universe and are therefore present in the haloes containing the Pop. III stars before these even form. If the PBHs do not comprise the entire DM, then the PBH mass could be larger than we have discussed heretofore, though contributing only a small fraction of the critical density.

As we will show below, the maximal accretion rate computed here is somewhat slower than the rate for the formation of a larger BH at the center of the star; all the effects combined thus lead to a big BH at the center.

3.3 Other mechanisms for energy release by PBHs

One may be also concerned about two other mechanisms in which PBHs can release energy: Hawking radiation (Hawking (1975,1976)) and positron emission (Bambi et al. (2008b)).

3.3.1 Hawking radiation

The luminosity due to Hawking radiation is maximal for the smallest mass BHs. We thus consider the (unrealistic) possibility that all the cosmological DM is made of PBHs with mass $M_{BH} = 10^{14}$ g. The Hawking luminosity per BH from γ, $e^{\pm}$ and $\mu^{\pm}$ emission is $7 \cdot 10^{18}$ erg/s (Page (1976)) and their total contribution to the power of a 10 $M_{\odot}$ star would be at the level of 10^{35} erg/s, roughly 2 or 3 orders of magnitude smaller than the ordinary stellar luminosity, $4 \cdot 10^{37}$ erg/s. If the mass of the star were 100 $M_{\odot}$, the relative contribution would be smaller, because the stellar luminosity increases by a factor 100, while the BH luminosity increases by a factor 10. Higher Hawking luminosity would demand smaller PBHs. However, if the PBHs had an initial mass $M_{BH} = 10^{13}$ g, their lifetime would be $\tau < 10^5$ yr, that is much shorter than the age of the Universe when first stars formed. Thus fusion luminosity always dominates over the Hawking radiation.

3.3.2 Schwinger effect

The second mechanism, positron production due to Schwinger effect at the BH horizon, has been recently discussed in Bambi et al. (2008b). Because protons are much more massive than electrons, it is much easier for BH to capture protons. Whereas the protons fall right into the BH, the electrons interact frequently via Compton scattering on their way into the BH and are prevented from falling in as easily. Hence the BH builds up a positive electric charge. For a BH mass $M_{BH} < 10^{20}$ g, the electric field at the BH horizon can exceed the critical value for vacuum stability, i.e. $E_c = m_e^2/e$, so that electron–positron pairs can be efficiently produced (Schwinger effect). Then, electrons are back–captured while positrons escape. The net result is to convert protons of the surrounding plasma into 150 MeV positrons. The accretion rate of protons is (Bambi et al. (2008b))

$$\dot{N}_p = 10^{30}\left(\frac{M_{BH}}{10^{20}\mathrm{g}}\right)^2\left(\frac{1\mathrm{keV}}{T}\right)^{3/2}\left(\frac{\rho_b}{1\ \mathrm{g/cm}^3}\right)\ \mathrm{s}^{-1}\,. \tag{14}$$

We note that mechanism is not the same as Bondi accretion, and that the expression above is not obtained from eq. (10). By contrast, Bondi accretion is the accretion of gas where particles collide with one another other, losing their tangential velocity but gaining radial velocity towards the star. This hydrodynamic approximation is applicable if the characteristic length scale is larger than the

mean free path of particles. Here, the size of the BH is smaller than the proton mean free path, λ_p, and we consider protons at distances $r < \lambda_p$ with small velocities, so they are gravitationally bound to the BH. These protons lose energy by bremsstrahlung or synhrotron radiation near the BH and in this sense they are not non-interacting. The picture is very much diffent from the hydrodynamical one and the calculations of the proton accretion rate can be found in Bambi et al. (2008b). Once equilibrium is reached between the accretion rate and the Schwinger discharge rate, the luminosity per BH is roughly (Bambi et al. (2008b))

$$L_{e^+} \sim 3 \times 10^{26} \left(\frac{M_{BH}}{10^{20}\ \mathrm{g}}\right)^2 \left(\frac{1\ \mathrm{keV}}{T}\right)^{3/2} \left(\frac{\rho_b}{1\ \mathrm{g/cm^3}}\right)\ \mathrm{erg/s}\,. \tag{15}$$

For $M_{BH} = 10^{20}$ g, this equation would then imply that the total Schwinger luminosity is roughly 10^{36} erg/s for a star of mass $10 - 100\ M_\odot$, which is comparable to the fusion luminosity for $10\ M_\odot$ stars given in Eq. (2) but far below the fusion luminosity for $100\ M_\odot$ stars given in Eq. (1). However this value of the Schwinger luminosity is never reached, because the rate for proton capture is $\sim 10^{29}\ \mathrm{s}^{-1}$ ($6 \times 10^{30}\ \mathrm{s}^{-1}$) for a $100\ M_\odot$ ($10\ M_\odot$) star, while the rate to create the e^+/e^- pairs is the product of the production rate per unit volume, $\sim m_e^4$, and the volume of the region around the BH in which the electric field exceeds the critical value E_c. The latter is a spherical shell of thickness about $1/m_e$, so the volume turns out to be r_g^2/m_e, where $r_g = 2G_N M_{BH}$ is the BH gravitational radius. The pair production rate is $\sim m_e^3 r_g^2 \sim 5 \times 10^{26}\ \mathrm{s}^{-1}$ for a 10^{20} g BH (the Schwinger discharge is fastest for this BH mass). Hence the equilibrium between the capture and the Schwinger mechanism is reached for a Schwinger luminosity that is several orders of magnitude lower than given above for the stellar mass $M_* = (10 - 100)\ M_\odot$. Thus fusion always dominates over the Schwinger effect as a heat source.

4 Formation of a larger black hole at the center of the star via Dynamical Friction

4.1 Main Sequence Star

The most important phenomenon associated with the PBHs inside the first stars is the formation of a larger BH at the center. It is well known that gravitational interactions cause every heavy body moving into a gas of lighter particles to lose energy by dynamical friction (Binney & Tremaine (1987)). Thus, PBHs inside a star are expected to sink to the center of the star, eventually forming one single large BH.

We will use Chandrasekhar's dynamical friction formula to compute the timescale for the PBHs to sink to the center of the star. If we assume that the gas of light particles has a Maxwellian velocity distribution with dispersion σ, then the deceleration of a BH moving at a velocity v_{BH} with respect to the rest frame of the fluid is

$$\frac{d}{dt}\vec{v}_{BH} = -4\pi\, G_N^2\, M_{BH}\, \rho_b\, \ln\Lambda\, \frac{\vec{v}_{BH}}{v_{BH}^3} \left[\mathrm{erf}(X) - \frac{2X\exp(-X^2)}{\sqrt{\pi}}\right] \tag{16}$$

where $X \equiv v_{BH}/(\sqrt{2}\sigma)$, erf is the error function, ρ_b is the density of the background particles and $\ln\Lambda \approx \ln(M_*/M_{BH})$ is the Coulomb logarithm [4]. There are two possible regimes, depending on whether v_{BH} is larger or smaller than the velocity dispersion inside the star, $\sigma = \sqrt{T/m}$ (Binney &

[4]The actual definition of Coulomb logarithm is (see Binney & Tremaine (1987))

$$\ln\Lambda = \ln\frac{b_{max}\,\sigma^2}{G_N(M_{BH} + m)}\,,$$

where b_{max} is the maximum impact parameter, σ^2 is the mean square velocity of the gas and m the molecular weight. Numerical simulations show that b_{max} can be assumed of order the orbital radius of the object, say R. Since $\sigma^2 \sim G_N M_*(R)/R$, a reasonable estimate of Λ is $M_*(R)/M_{BH}$.

Tremaine (1987)). Here T is the local gas temperature and m is the molecular weight. The factor in the square brackets

$$F(v_{BH}) = \mathrm{erf}(X) - \frac{2X\exp(-X^2)}{\sqrt{\pi}} \tag{17}$$

tends to unity for $v_{BH} \gg \sigma$ and tends to $v_{BH}^3/2\sqrt{2\pi}\sigma^3$ for $v_{BH} \ll \sigma$.

The vector equation (16) can be rewritten as the following two scalar equations:

$$\ddot{r} = -\frac{M_*(r)G_N}{r^2} + \frac{J^2}{r^3} - \gamma(v_{BH})\dot{r}, \tag{18}$$

$$\dot{J} = -\gamma(v_{BH})J, \tag{19}$$

where r is the distance of BH form the star center, $J = r^2\dot{\theta}$ is the BH angular momentum per unit mass, θ is the azimuth angle, $v_{BH} = \sqrt{\dot{r}^2 + J^2/r^2}$,

$$M_*(r) = \int_0^r d^3r\rho_b(r) \tag{20}$$

is the stellar mass inside radius r, and

$$\gamma(v_{BH}) = 4\pi G_N^2 M_{BH}\rho_b \ln\Lambda \, \frac{\mathrm{erf}(X) - 2X\exp(-X^2)/\sqrt{\pi}}{v_{BH}^3}. \tag{21}$$

Since the characteristic gravitational time scale

$$\tau_g = \sqrt{\frac{r^3}{M_*(r)G_N}} \sim \left(\frac{3}{4\pi\rho_b G_N}\right)^{1/2} \approx 1900\left(\frac{1\ \mathrm{g/cm^3}}{\rho_b}\right)^{1/2} \mathrm{s} \tag{22}$$

is much shorter than the lower limit on the characteristic dynamical friction time scale

$$\tau_{DF} = \frac{\sigma^3}{4\pi\, G_N^2 M_{BH}\rho_b \ln\Lambda} \approx 5\cdot 10^{10}\left(\frac{10^{24}\mathrm{g}}{M_{BH}}\right)\left(\frac{\sigma}{3\cdot 10^7\ \mathrm{cm/s}}\right)^3\left(\frac{1\ \mathrm{g/cm^3}}{\rho_b}\right)\left(\frac{10}{\ln\Lambda}\right)\mathrm{s}\,, \tag{23}$$

we can approximately solve eqs. (18, 19) as follows[5]. We may neglect the last term in the r.h.s. of eq. (18) and assume approximate equality $J^2 \approx G_N M_*(r)r$. Using this result we can integrate eq. (19), which now takes the form:

$$\dot{v}_{BH} = -\frac{\sigma^3 F(v_{BH})}{v_{BH}^3\tau_{DF}}\, v_{BH}\,, \tag{24}$$

and calculate the time of capture of small BHs at the stellar center. The result depends upon the initial velocity of the BH which we may estimate assuming that the BH is on a circular orbit of radius r determined by the stellar mass $M_*(r)$ interior to radius r, i.e., $v_{BH} = \sqrt{G_N M_*(r)/r}$. We find that, in the outer regions of the star, $v_{BH} \gtrsim \sigma$, in which case eq. (24) scales as $\dot{v}_{BH} = -\sigma^3/(\tau_{DF}v_{BH}^2)$. In the inner regions of the star, near the stellar center, we find the opposite limit of $v_{BH} \lesssim \sigma$, in which case eq. (24) scales as $\dot{v}_{BH} = -v_{BH}/(2\sqrt{2\pi}\tau_{DF})$ instead. Thus, in the latter case, the time of BH formation is about

$$\begin{aligned} \tau_f &\approx 2\sqrt{2\pi}\tau_{DF}\ln\left(\frac{v_{BH}^{in}}{v_{BH}^{f}}\right) \approx 2\sqrt{2\pi}\tau_{DF}\ln\left(\frac{R_{in}}{R_f}\right) \approx \\ &\approx 1.4\cdot 10^4\left(\frac{10^{24}\ \mathrm{g}}{M_{BH}}\right)\left(\frac{\sigma}{3\cdot 10^7\ \mathrm{cm/s}}\right)^3\left(\frac{1\ \mathrm{g/cm^3}}{\rho_b}\right)\left(\frac{10}{\ln\Lambda}\right)\mathrm{yr}\,, \end{aligned} \tag{25}$$

[5]The reader might also be concerned whether we can neglect the third term in eq. (18) when considering the opposite limit as v_{BH} goes to zero. In this case, σ goes to v_{BH} in eq. (23). Again, the third term in eq. (18) is completely negligible and even more so than when eq. (23) depended upon σ.

where v_{BH}^{in} is the initial PBH velocity, so $v_{BH}^{in} \approx \sigma$ and implies $R_{in} \sim 10^{10}$ cm, while v_{BH}^{f} is the final PBH velocity, when $R_f = 4 \cdot 10^2$ cm, that is, when the orbit of the PBH is equal to the Schwarzschild radius of the final BH. In the case $v_{BH} \gtrsim \sigma$, the timescale becomes

$$\tau_f \approx \frac{\tau_{DF}}{\sigma^3} \left(\frac{1}{v_{BH}^{f}} - \frac{1}{v_{BH}^{in}} \right) , \tag{26}$$

which can be quite a bit longer than the one for the case $v_{BH} \lesssim \sigma$ for v_{BH}^{in} moderately larger than v_{BH}^{f}. As shown later, this is not a problem, because we always have a sufficient number of PBHs at small radii, where $v_{BH} \lesssim \sigma$.

The case of very eccentric orbits does not significantly change the picture. A simple estimate can be obtained assuming radial motion and constant matter density ρ_b. In the absence of dynamical friction, the motion of the PBHs can be treated as an harmonic oscillator with period τ_g and velocity $\sim (R_0/\tau_g)\cos(t/\tau_g)$, where R_0 is the maximum distance from the center of the star and t is the time. Since the maximum velocity exceeds $3 \cdot 10^7$ cm/s for $R_0 > R_* \sim 10^{11}$ cm, the equation of motion of PBHs inside the radius R_* is basically

$$\ddot{r} \approx -\frac{\dot{r}}{\tau_{DF}} - \frac{r}{\tau_g^2} \tag{27}$$

The differential equation is that of an underdamped harmonic oscillator:

$$r(t) \sim e^{-t/2\tau_{DF}} \cos(\omega t + \delta) \, , \tag{28}$$

where $\omega \approx 1/\tau_g$. We find $\tau_f = 2\tau_{DF} \ln\left(\frac{R_{in}}{R_f}\right)$, a timescale which is actually shorter than in the circular case. Thus we expect that the result in eq. (25) is a reasonable estimate of the timescale.

To obtain more accurate quantitative estimates of the dynamical friction timescale on which the PBHs sink to the center of the star, we did numerical calculations assuming that the star can be modeled as an $n = 3$ polytrope, which is known to roughly reproduce the stellar properties of a star dominated by radiation pressure. We can then obtain density and temperature profiles for a 100 $M_\odot$ star which are plotted in fig. (1). If one does the full stellar structure of a star of a Pop. III star, the exact answer is different than found assuming a polytrope. The difference is on the order of at most tens of a percent.

Subsequently, we can now compute the timescale for the case of $M_{BH} = 10^{24}$ g; luckily, The resultant timescale can easily be scaled to other BH masses since it is inversely proportional to M_{BH}. To be specific, we investigated the case of a 100 $M_\odot$ star. We found that the transition from fast to slow BH velocity (relative to gas particle velocity) takes place at $R_c \sim 2 \cdot 10^{10}$ cm. As explained above, the dynamical friction for BH outside of this radius is proportional to $1/v_{BH}^2$, while, for smaller radii it is proportional to v_{BH}. Roughly 50% of the BHs are initially inside the radius $R_i = 1.4 \cdot 10^{11}$ cm; these BH take $1 \cdot 10^4$ yr or less to sink to R_c. (We have also computed the timescales for infall for BH coming in from different initial radii R_i to the same value of R_c; our results are shown in Table (1). Subsequently each BH takes another $\sim 5 \cdot 10^4$ yr to sink from R_c to $R_f = 4 \cdot 10^2$ cm. The latter is the Schwarzschild radius of the final BH at the center of star. Thus the timescale for half of the PBHs to form a single large BH at the center of the 100 $M_\odot$ star is roughly

$$\tau_f = 6 \cdot 10^4 \left(\frac{10^{24}\ \mathrm{g}}{M_{BH}} \right) \ \mathrm{yr}. \tag{29}$$

Thus for $M_{BH} > 10^{22}$ g, in a 100 $M_\odot$ star, the timescale for the formation of a large central BH is less than a million years, which can have a significant impact on the evolution of the star. We note that, once the central BH mass is $\sim 10^{25}$g, the (fastest possible) accretion timescale in Eq. (13) becomes comparable to the dynamical friction timescale Eq. (29); the result of both effects is a single large BH inside the star.

If the mass of the star is 10 $M_\odot$, the sinking time is not significantly different.

R_i (cm)	$M_*(R_i)/M_*(R_*)$	$\rho(r_i)$ (g/cm^3)	Time (yrs)
$4.7 \cdot 10^{11}$	1.0	0.5	205,000
$2.4 \cdot 10^{11}$	0.9	3.1	30,000
$1.4 \cdot 10^{11}$	0.5	8.7	11,000
$6.5 \cdot 10^{10}$	0.1	17.3	5,000

Table 1: Numerical results of the timescale for BHs to move from a variety of initial radii R_i to a smaller radius $R_c \sim 2 \cdot 10^{10}$ cm. $M_*(R_i)/M_*(R_*)$ is the fraction of the mass of the star inside the radius R_i, which is equal to the initial fraction of PBHs inside the radius R_i. $\rho(R_i)$ is the stellar density at R_i. The mass of the BH has been fixed to 10^{24}g

.

Additional PBHs from outside the star may also fall onto the central BH via dynamical friction. For a baryon density profile that scales as $\rho_b(r) \propto r^{-2.3}$ outside the star, we find that the dynamical friction timescale is

$$\tau_{DF} = 2 \times 10^{16}\mathrm{yr} \left(\frac{\ln\Lambda}{10}\right) \left(\frac{r_i}{1\mathrm{pc}}\right)^{1.85} \left(\frac{M_{BH}}{10^{24}\mathrm{g}}\right)^{-1}, \tag{30}$$

where r_i is the initial radius of the infalling PBH and this equation has been computed in the fast BH regime with $\dot{v}_{BH} \propto v_{BH}^{-2}$. Thus it takes a very long time for dynamical friction to be effective at pulling in BH from typical radii in the minihalo. From closer in, the timescale can be significantly shorter, e.g., it takes 150,000 years for a 10^{24} g PBH to go from 3×10^{12} cm (~ 10 times the radius of the star) to the center. However, the amount of mass in PBHs inside this radius is 2.4×10^{29} g, more than an order of magnitude less than the amount already in the star from Eq. (6), and is therefore negligible. Thus dynamical friction does not pull in significant mass in PBHs from outside the star.

4.2 Protostellar Phase

One may ask whether dynamical friction is already effective during the protostellar phase, long before the Pop. III star comes to exist on the main sequence. Early on, there is a collapsing molecular cloud which is very diffuse and becomes more and more dense as it cools via molecular hydrogen cooling. The protostellar clouds stop collapsing once they become protostar nuggets with 10^{-2} $M_\odot$ in mass, hydrogen densities of 10^{21} cm^{-3}, and radii $\sim 5 \times 10^{11}$ cm (Yoshida et al. (2008)). In the standard picture of Pop. III star formation, there is then accretion onto these nuggets until the stars reach ~ 100 $M_\odot$ and go onto the main sequence.

Can the PBHs already sink to the center during this earlier phase and cause the protostar to go directly to a BH, avoiding the main sequence phase altogether? Inside the protostar, the appropriate regime for dynamical friction is that of slow BH, with $\dot{v}_{BH} \propto v_{BH}$. Such protostellar clouds have much lower densities than the subsequent Pop. III stars on the main sequence, and consequently are ineffective at causing the PBHs to slow down via dynamical friction. We have checked that the timescale is simply too long for PBHs to play any role during the collapse of the protostellar cloud, unless the PBHs are much more massive than have been considered in this paper. However, once the nugget forms, the baryon density is high enough to trap PBHs of mass $> 10^{26}$ g with dynamical friction. At this point the Kelvin-Helmholtz time $\sim \tau_{DF} \sim 10$ yr. The amount of DM (PBHs) inside the nugget is $\sim 10^{28}$ g, so that the initial central BH is only this big. However, it quickly eats the rest of the 10^{-2} $M_\odot$ protostar, and presumably can grow at least to the value of the original 1000 $M_\odot$ Jeans mass of unstable material.

5 Eating Pop. III stars

We have shown that the PBHs can sink to the center of the star and form a single larger BH in a reasonable timescale (for $M_{PBH} > 10^{22}$ g) to change the evolution of the star. We now need to address the subsequent fate of the star: can the BH really accrete at the Bondi rate and swallow the

whole star quickly? Alternatively, does the radiation pressure from the accretion luminosity slow down the accretion rate and make the star have a normal evolution? In general, the accretion of matter onto an object with a solid surface (e.g. a neutron star) is limited by the radiation produced by the accreting gas,

$$L_a = \eta \dot{M} \tag{31}$$

where η is basically the gravitational potential per unit mass on the surface of the object. Nevertheless, in the case of accretion onto BH, the picture is more complex and the phenomenology richer. If the cooling mechanism is efficient, the gravitational energy of the accreting gas is radiated away and the gas temperature is much smaller than the local virial temperature. This case is similar to the one involving objects with a solid surface: η is equal to the binding energy per unit mass at the Innermost Stable Circular Orbit (ISCO), since we presume that the gas inside the ISCO falls quickly into the BH and is unable to emit further radiation. So, for Schwarzschild BHs $\eta = 0.057$, while for Kerr BHs the efficiency parameter can be as high as 0.42 (Shapiro & Teukolsky (1983)). On the other hand, if the cooling is not efficient, the gravitational energy is stored as thermal energy into the gas rather than being radiated. That can occur if the gas density is very low and particles do not scatter each other very much, or in the opposite case, when the medium is optically thick and radiation is trapped, as happens for high accretion rate. Here, unlike neutron stars, BHs have an event horizon and the energy can be lost into the BH. η turns out to be very small and the accretion luminosity can be low. The accretion rate of matter can thus be high.

We will argue that the BHs at the center of the first stars may accrete at the Bondi rate, with the star adjusting to keep the luminosity equal to the Eddington value, corresponding to a small value of η in Eq. (31). With Bondi accretion, the BHs can swallow the star in a short time, becoming 10–1000 $M_\odot$ BHs. There is considerable discussion of BHs accreting material inside stars in the literature. We present here some of the possibilities for the evolution of these objects. In all cases, the end result is a 10–1000 $M_\odot$ BH. In the case of radiative stars, we may follow Begelman (1978), where the author discusses the steady flow accretion onto a Schwarzschild BH of a non–relativistic gas where the radiation pressure at infinity is much larger than the particle pressure and the radiation–particle coupling is provided by the Thomson scattering. The medium is optically thick at all the relevant scales and radiation is transported by diffusion and convection. Here one finds a trapping surface at the radius

$$R_t = \frac{\dot{M}_{BH}\, \sigma_{Th}}{4\pi m_p}\,, \tag{32}$$

inside which the radiation is convected inward and swallowed by the BH faster than it can escape to infinity.[6] If R_t is much larger than the Bondi radiu $R_B = 2GM/v^2$, then the radiation is effectively trapped, that is it is convected inwards faster than it can diffuse outwards. In our case, using eq(10),

$$\frac{R_t}{R_B} = 6 \cdot 10^4 \left(\frac{M_{BH}}{10^{30}\ \mathrm{g}}\right) \left(\frac{\rho_b}{1\ \mathrm{g/cm^3}}\right) \left(\frac{v}{3 \cdot 10^7\ \mathrm{cm/s}}\right)^{-1} . \tag{33}$$

Given the typical densities and temperatures inside Pop. III stars, this condition is verified, the process is essentially adiabatic and in principle the BH is capable of accreting at an arbitrary high rate. Since radiation is trapped, the luminosity produced by the accretion process can not exceed the Eddington value, and the radiative efficiency effectively adjusts in order to keep $L \sim L_{Edd}$ (Begelman (1978)). As long as accretion is spherical, with zero angular momentum, the central PBH can accrete ad libitum, and eventually swallow the whole star. In the presence of limited angular momentum we can argue that as long as the accretion disk that forms is all contained within the trapping radius, then radiation remains trapped and the growth of the BH can continue (Volonteri & Rees (2005)). We can take as a safe limit the condition that the disc is all within the trapping radius; this provides a lower limit to when accretion stops. The outer edge of the accretion disk, R_D, is roughly where the specific angular momentum of the gas equals the angular momentum of a gas element in a Keplerian

[6] Clearly R_t cannot be larger than the radius of the star, $R_\star$. In this case, we take $R_t = R_\star$.

circular orbit, therefore:

$$\frac{R_D}{R_B} = \sqrt{2}\left(\frac{V(R_B)}{c_s}\right)^2 \tag{34}$$

where c_s is the sound speed and $V(R_B)$ is the rotational component of the velocity at the Bondi radius. In this case it still seems possible that the radiative efficiency drops so that the BH can accept the material without greatly exceeding the Eddington luminosity. Relaxing the assumptions of zero angular momentum and absence of mechanical turbulence and/or magnetic fields, the actual matter accretion rate presumably decreases, but the evolution of the star is slowed down as well. On the other hand, for very high angular momenta, it sounds reasonable that the system looks like a collapsar (MacFadyen & Woosley (1999)).

Convective stars: 100 $M_\odot$ Pop. III stars are primarily convective (Heger et al. (2007)). One can compute the Eddington luminosity in the case of a BH inside a mostly convective star with a radiative outer envelope as follows (Begelman et al. (2008)). There is no radiation pressure inside the convective zone, so the luminosity from the BH can easily get to the radiative outer envelope. Out there radiation pressure does exist. Then the Eddington luminosity at this outer region (which basically contains the entire star) is the relevant quantity. In short, one should use the Eddington luminosity of the star rather than Eddington luminosity of the BH, which means substituting M_* for M_{BH} in Eq. (8). Doing this, one finds

$$L_{BH} = L_{E,*} = 10^{40}\ \mathrm{erg/sec}\ (M_*/100 M_\odot)\,. \tag{35}$$

This value is significantly larger than the numbers obtained in Eq. (8) because it is the Eddington luminosity of the star rather than that of the BH. Here the accretion luminosity is bigger than the fusion luminosity. The consequence for the star will be that it must expand, it will cool, and fusion will shut off. At that point the star looks like the quasistars in Begelman et al. (2008). These authors have worked out the stellar structure for a BH of arbitrary mass inside a star of arbitrary mass, where the only heat source is accretion luminosity. These authors were studying a different problem: they were not looking at Pop. III stars in 10^6 $M_\odot$ haloes; instead they were looking at cooler regions of similar content in 10^7 $M_\odot$ haloes. Although the context was different, the resultant objects should be very similar.

There are 2 possibilities for the accretion: 1) The accretion may be spherical. In that case η can be very small, as discussed in Bisnovatyi-Kogan & Lovelace (2002). There is no problem having $\eta = 10^{-6}$ so that the Eddington luminosity in Eq. (35) is compatible with Bondi accretion at $\dot{M} = 10^{46}$ erg/sec. Then it takes a thousand years to swallow the 100 $M_\odot$ star (see Eq. (53) in Begelman et al. (2008)). Even more interesting is to contemplate the possibility that the star is accreting further mass from the halo outside it, e.g. at a rate 0.01 $M_\odot$/ year (McKee & Tan (2007))[7]. Then the BH can end up very large as seen in Eq. (52) of Begelman et al. (2008), possibly eating all 10^5 $M_\odot$ of baryons in the DM halo.

2) The accretion may be in a disk. If the disk is thin and radiatively efficient, then $\eta \sim 0.1$ and $\dot{M} \ll \dot{M}_B$ (the accretion rate is much slower than Bondi). However, in different geometries, η can become much smaller (Abramowicz & Lasota (1980)). Begelman et al. (2008) claim that the accretion stops once you hit "the opacity crisis." This happens when the photospheric temperature (at the edge of the star) goes down to a critical value, so that the radiation pressure in the outer envelope vanishes, nothing prevents the star from going super–Eddington and blowing off all its gas. This leaves behind an exposed BH that no longer accretes anything. They find that for a fixed stellar mass of 100 $M_\odot$, the resultant object is a 10 solar mass BH in 10^7 years, but nothing bigger, due to this opacity crisis. On the other hand, if the star is accreting further material from the outside, then you can end up with a 400 $M_\odot$ BH if the accretion rate of material onto the star is 10^{-2} $M_\odot$/ yr (McKee & Tan (2007)) or 4000 $M_\odot$ BH if the accretion rate onto the star is 10^{-1} $M_\odot$/ yr. Again, it takes 10^7 years to reach this. In the meantime, during this 10^7 years, you have a "PBH Dark (matter

[7]The accretion rate for Pop. III stars is still highly uncertain, and certainly variable as a function of time. The values we quote are higher than typical estimates for prolonged accretion rate (see e.g. Figure 8 of McKee & Tan (2008)), but still definitely possible, especially if PBHs somewhat reduce feedback effects.

powered) Star", i.e. a Pop. III star powered by accretion luminosity rather than by fusion. The exact accretion rate is none the less quite uncertain. Convective energy transport is itself limited and bounds the accretion rate (Begelman et al. (2008))

$$\dot{M}_{BH} \lesssim \frac{\dot{M}_B c_s^2}{\eta}\,. \tag{36}$$

Since $c_s \sim 10^{-3} - 10^{-2}$, the actual accretion rate might be much smaller than the Bondi rate $\dot{M}_B$, unless η is quite small, say $\eta < 10^{-4} - 10^{-6}$. This is not a problem for spherical accretion, but might affect results for disk accretion. Regardless, this will require more study. Even accretion onto the first stars without the additional effects from primordial black holes is presently still an unsolved problem.

We have argued that the BHs at the center of the first stars may accrete at the Bondi rate, so that the BHs can swallow the star in a short time, becoming 10–1000 $M_\odot$ BHs. This mechanism may produce the seeds to generate the super–massive BHs which have been observed even at high redshifts and at the centers of galaxies.

6 Summary and conclusions

Primordial black holes in the mass range $M_{PBH} \sim 10^{17} - 10^{26}$ g are viable dark matter candidates. They may be produced in the early Universe by many mechanisms and so far there are no constraints on their possible abundance. Assuming that they make part of the cosmological dark matter, we expect that due to dynamical friction primordial black holes will make up a small but significant mass fraction of the first stars. Primordial black holes with masses smaller than about 10^{22} g do not have a significant effect on the evolution of primordial stars, because their timescales for Bondi accretion and for dynamical friction are larger than the lifetime of a Main Sequence star of $10 - 100\ M_\odot$. On the contrary, primordial black holes heavier than 10^{22} g might sink quickly to the center of the star by dynamical friction and form a larger black hole, which could swallow the whole star in a short time. So, Pop. III stars would likely have lived for a short time, with implications for the reionization of the Universe after the cosmic dark ages and the nature of the first supernovae; in fact they may preclude any supernovae from the first stars. Although the BH swallowing the star shortens the star's lifetime and its contribution to reionization, the newly formed hole can become a new, alternative source of ionizing photons. The 10–100 $M_\odot$ BHs that form by swallowing the Pop. III stars may grow even larger: they reside in 1000 $M_\odot$ of gas that are in excess of the Jeans mass and may fall into the BH. Black holes of mass 1-1000 $M_\odot$ may result.

Depending on the accretion mechanism at this point, the black hole may accrete more matter and grow larger. The $10^6\ M_\odot$ minihaloes of dark matter contain $\sim 10^5\ M_\odot$ of baryonic matter. This accretion from the minihalo, as well as from other haloes merging with the one containing the black hole, would be from low density gaseous material ($\rho \sim 10^{-24}$ g/cm^3), which is considerably different from the accretion we considered earlier from within the star ($\rho \sim 1$ g/cm^3). In the case of accretion from the low density gas outside the star, feedback may become important. As we have shown, the timescale for the Pop. III stars to become black hole can be much shorter than the lifetime of the Pop. III stars (3 Myr for a 100 $M_\odot$ star), so that the feedback due to stellar heating and ionization of the medium surrounding the black hole may be minimal. However, the accretion may well be in a disk, with the accompanying radiation pressure as well as radiative feedback due to the accretion (Silk & Rees (1998); Springel et al. (2005); Ciotto & Ostriker (2001); Li et al. (2007); Pelupessy et al. (2007); Alvarez et al. (2008)) limiting the accretion rate. A recent study (Alvarez et al. (2008)) of the radiative feedback from the black hole accretion has been done for the case of $\eta = 0.1$ and a Pop. III star that has undergone its full lifespan, and finds reduced accretion onto the BH; the story may be different here. We have not studied these later stages. Since the end–products are $10 - 10^5\ M_\odot$ black holes, these objects may serve as seeds for Intermediate Mass Black Holes; the super–massive black holes which have been seen already at high redshifts (Haiman & Loeb (2001); Volonteri & Rees (2006)) and may be the progenitors of the super–massive black holes which are in the center of every normal galaxy today.

Even if the primordial black holes do not explain the entirety of the dark matter in the Universe, they may still play a role in the first stars. Heavier primordial black holes than the ones studied here, i.e., primordial black holes with $M_{BH} > 10^{26}$ g, are observationally constrained to be only a fraction of the total dark matter in the Universe, and yet could be important in the first stars. It would only take one such black hole to be pulled into the star via dynamical friction (timescale $\sim 10^7$ yr for a 1 $M_\odot$ black hole to get from 1 pc out into the center of the star (see Eq. (30)) and to quickly eat up the whole star. In fact, a single massive primordial black hole would already have a major effect during the protostellar phase while the molecular cloud is collapsing down into a protostar: the molecular cloud would already collapse into a black hole. In this case the fusion phase of a Pop. III star would be completely avoided. Another possibility would be to have the dark matter consist primarily of Weakly Interacting Massive Particles (WIMPs) but with a small component of primordial black holes. In that case there would be dark stars powered by WIMP annihilation (Spolyar et al. (2008)), which would become black holes once the primordial black holes sink to the center of the dark star.

In principle, if the effects described in this paper do not take place, one could place bounds on the black hole abundances of various masses. For example, if primordial black holes swallowed primordial stars too quickly, the cosmological metal enrichment would be problematic and in absence of viable alternatives, the current allowed mass range $M_{PBH} \sim 10^{17} - 10^{26}$ g could be further reduced to $\sim 10^{17} - 10^{22}$ g.

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References

Abel T., Bryan G. L. & Norman M. L., 2002, Science, 295, 93
Abramowicz M. A. et al. 2008, arXiv:0810.3140
Abramowicz M. A. & Lasota J. P., 1980, Acta Astron., 30, 35
Alcock C. et al. 2000, ApJ, 542, 281
Alvarez M. A., Wise J. H. & Abel T., 2008, arXiv:0811.0820
Bambi C., Dolgov A. D. & Petrov A. A., 2008a, PLB, 670, 174
—, 2008b, arXiv:0806.3440
Barkana R. & Loeb A., 2001, Phys. Rep., 349, 125
Begelman M. C., 1978, MNRAS, 184, 53
Begelman M. C., Rossi E. M. & Armitage P. J., 2008, MNRAS, 387, 1649
Binney J. & Tremaine S., 1987, *Galactic Dynamics* (Princeton University Press, Princeton, USA)
Bisnovatyi-Kogan G. S. & Lovelace R. V. E., 2002, New Astron.Rev., 45, 663
Bondi H., 1952, MNRAS, 112, 195
Bromm V., Coppi P. S. & Larson R. B., 2002, ApJ, 564, 23
Bromm V. & Larson R. B., 2004, ARAA, 42, 79
Carr B. J., 2003, Lect. Notes Phys., 631, 301
Carr B. J. & Hawking S. W., 1974, MNRAS, 168, 399
Carr B. J. & Sakellariadou M., 1999, ApJ, 516, 195
Ciotti L. & Ostriker J. P., 2001, ApJ, 551, 131
Crawford M. & Schramm D. N., 1982, Nature, 298, 538
Dokuchaev V. I., Eroshenko Yu. N. & Rubin S. G., 2007, arXiv:0709.0070
Dolgov A. D., Kawasaki M. & Kevlishvili N., 2008, arXiv:0806.2986
Dolgov A. & Silk J., PRD, 1993, 47, 4244
Freese K., Spolyar D. & Aguirre A., 2008, JCAP 0811:014

Freese K., Gondolo P., Sellwood J., & Spolyar D. 2009, ApJ, .693, 1563
Gould A., 1992, ApJ, 386, L5
Haiman Z. & Loeb A., 2001, ApJ, 552, 459
Hawke I. & Stewart J. M., 2002, CQG, 19, 3687
Hawking S. W., 1971, MNRAS, 152, 75
—, 1975, Comm. Math. Phys., 43, 199
—, 1976, Comm. Math. Phys., 46, 206
—, 1989, PLB, 231, 237
Heger A. et al 2007, arXiv:0711.1195
Heger A. & Woosley S., private communication
Hollenbach D. & McKee C. F., 1979, ApJS, 41, 555
Jedamzik K., PRD, 1997, 55, 5871
Li Y. et al. 2007, ApJ, 655, 187
MacFadyen A. & Woosley S. E., 1999, ApJ, 524, 262
Marani G. F. et al. 1999, ApJ, 512, L13
Matsuda T., Sato H. & Takeda H., 1971, Prog. Theor. Phys., 46, 416
McKee C. F. & Tan J. C., 2008, ApJ, 681, 771
Navarro J. F., Frenk C. S. & White S. D. M. 1997, ApJ, 490, 493
Nemiroff R. J. & Gould A., 1995, ApJ, 452, L111
Novikov I. D. et al. 1979, A&A, 80, 104
Omukai K. & Nishi R., 1998, ApJ, 508, 141
Page D. N., 1976, PRD, 13, 198
Page D. N. & Hawking S. W., 1976, ApJ, 206, 1
Peebles P. J. E. & Dicke R. H., 1968, ApJ, 154, 891
Pelupessy F. I., Di Matteo T. & Ciardi B., 2007, ApJ, 665, 107
Polnarev A. & Zembowicz R., 1991, PRD, 43, 1106
Ripamonti E. & Abel T., 2005, arXiv:astro-ph/0507130
Rubin S. G., Khlopov M. Y. & Sakharov A. S., 2000, Grav. Cosmol., S6, 51
Sellwood J. A. & MaGaugh S. S., 2005, ApJ, 634, 70
Seto N. & Cooray A., 2004, PRD, 70, 063512
Shapiro S. L. & Teukolsky S. A., 1983, *Black Holes, White Dwarfs, and Neutron Stars* (John Wiley, New York, USA)
Silk J. & Rees M. J, 1998, A&A , 331, L1
Spolyar D., Freese K. & Gondolo P., 2008, PRL, 100, 051101
Springel V., Di Matteo T. & Hernquist L., 2005, MNRAS, 361, 776
Tegmark M. et al. 1997, ApJ, 474, 1
Tisserand P. et al. 2007, A&A, 469, 387
Volonteri M. & Rees M. J., 2005, ApJ, 633, 624
—, 2006, ApJ, 650, 669
Yoshida N., Omukai K. & Hernquist L., 2008, Science, 321, 669
Yoshida N., Sugiyama N. & Hernquist L., 2003, MNRAS, 344, 481
Zeldovich Ya. B. & Novikov I. D., 1966, Sov. Astron., 10, 602

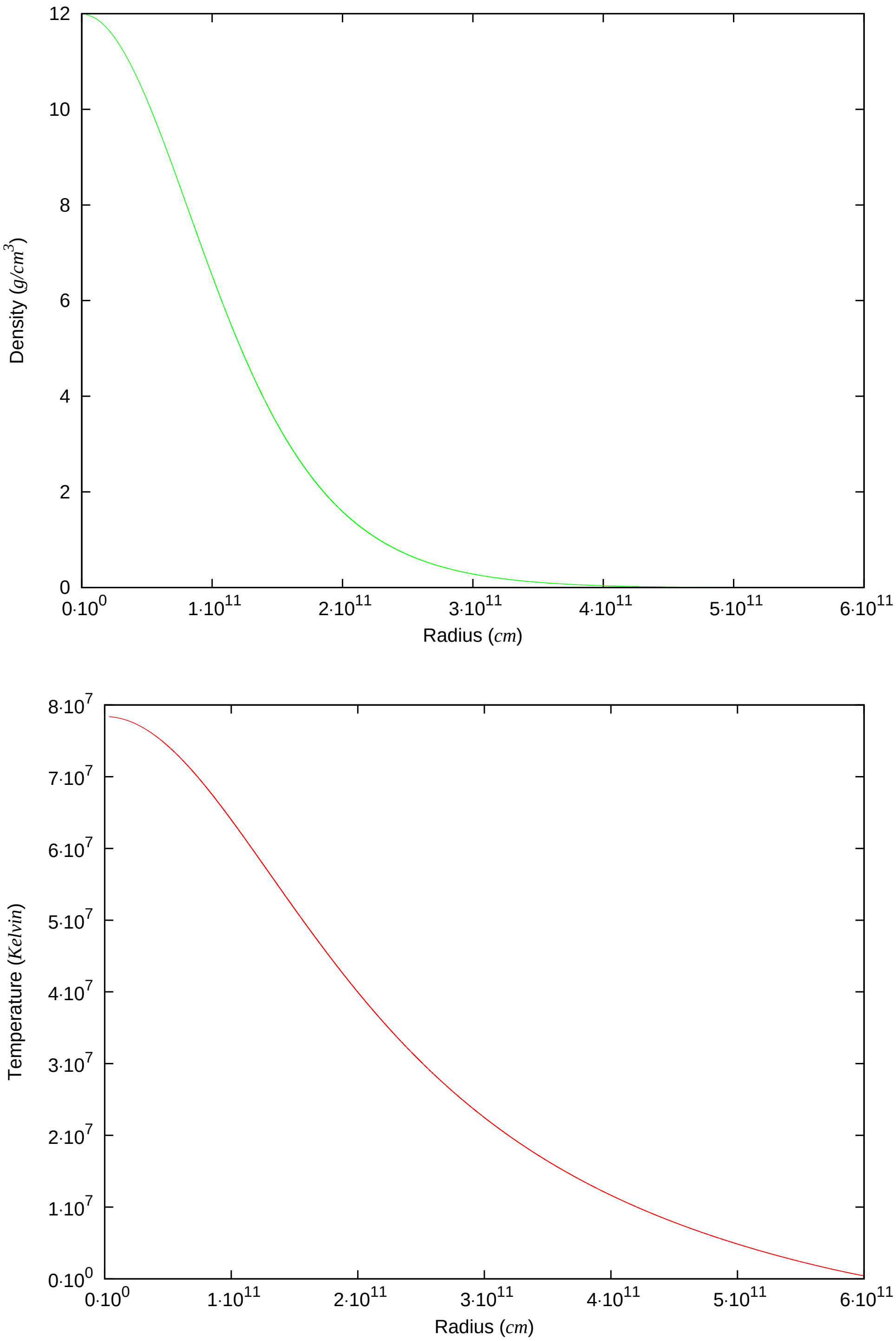

Figure 1: Density (top panel) and temperature (bottom panel) profile for the $n = 3$ polytrope star of mass 100 $M_{\odot}$ used in our simulations.